Lower Bound Techniques in Distributed Computing: Home Assignment 1

April 3, 2006
Due: April 24, 2006.

1. We have proved a $\frac{n-1}{n}u$ lower bound on the clock skew achievable in a clock synchronization algorithm. The lower bound was proved assuming that the communication graph is a clique and the message delay on every communication link is in $[d-u, d]$.

   Extend this lower bound to a system where the communication graph is arbitrary and the message delay on communication link e is in $[d-u_e, d]$, in either direction.

   Hint: Consider first the case in which the communication graph is a clique, but uncertainties are non-uniform.

2. In the ALL-AWAKE problem, a process that does not fail terminates within a finite number of steps and returns either 0 or 1. If every process terminates, then at least one process returns 1. No process returns 1 unless every process has taken at least one step.

   (a) Prove that in an asynchronous shared memory system of n processes that communicate using load-link/store-conditional objects, the worst case step complexity of ALL-AWAKE is in $\Omega(\log n)$.

   (b) Prove that, for any implementation of a counter (an object which only supports the operation fetch&increment) shared by n processes that communicate using load-link/store-conditional objects, a process requires $\Omega(\log n)$ steps in the worst case to perform a fetch&increment operation.

   A load-link/store-conditional object is linearizable and supports two operations:

   - *LL*, which returns the value of the object, and
   - *SC(v)*, which either returns true and changes the value of the object to v or returns false and leaves the value of the object unchanged.

   If a process p performs *SC*, the it returns true if and only if no other process has performed an *SC* operation that returned true since p last performed *LL*.